



An experimental study of heat transfer by laminar natural convection between an electrically-heated vertical plate and both Newtonian and non-Newtonian fluids

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Abstract

Natural convection from a vertical electrically-heated plate (90 mm × 79 mm) to both Newtonian and non-Newtonian fluids has been studied experimentally for conditions of constant surface heat flux. For Newtonian fluids, a very wide range of viscosities (10 000-fold), of densities (1000-fold), and of Prandtl numbers (2500-fold) has been covered. The effect of shear-thinning non-Newtonian behaviour has been investigated using fluids with power-law indices from 0.48 to 0.81. Over this very wide range of conditions, the measured vertical temperature profile at the surface of the plate, and its dependence on the heat flux, has been found to conform well to theoretical predictions. The values of the coefficients in the well-established relations between Nusselt, Grashof and Prandtl numbers have then been evaluated and they have been shown to compare well with theoretical predictions and experimental results of other workers. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

A function of physical properties defined by equation (10)
 A_{NN} function of physical properties and rheology defined by equation (20)
 B function of physical properties defined by equation (11)
 B_{NN} function of physical properties and rheology defined by equation (21)
 C coefficient in equation (7) for Newtonian fluid
 C_{NN} coefficient in equation (17) for power-law fluid
 C_p specific heat capacity of fluid at constant pressure [J kg⁻¹ K⁻¹]
 c coefficient in equation (3) for Newtonian fluid
 c_{NN} coefficient in equation (14) for power-law fluid
 g acceleration due to gravity [m s⁻²]
 Gr_x Grashof number defined by equation (2)
 Gr_x^* Grashof number defined by equation (5)

Gr_{xNN} Grashof number defined by equation (13) for power-law fluid
 Gr_{xNN}^* Grashof number defined by equation (15) for power-law fluid
 K consistency coefficient for power-law fluid [Pa s ^{n}]
 k thermal conductivity of fluid [W m⁻¹ K⁻¹]
 L total length of plate in vertical direction [m]
 n flow behaviour index for power-law fluid
 Nu_x Nusselt number, $q_s x / k \Delta T$ (constant ΔT)
 Nu_x^* Nusselt number, $q_s x / k \Delta T$ (constant q_s)
 Pr Prandtl number for Newtonian fluid
 Pr_{NN} Prandtl number for power-law fluid defined by equation (12)
 q_s heat flux at surface [W m⁻²]
 ΔT temperature difference between surface and bulk fluid [K]
 x distance from leading edge of surface [m].

Greek symbols

β coefficient of cubical expansion of fluid [K⁻¹]
 μ viscosity of fluid [Pa s]
 ρ density of fluid [kg m⁻³]
 τ shear stress [Pa]
 $\dot{\gamma}$ shear rate [s⁻¹].

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1. Introduction

In a recent paper [1], results were reported of a study of heat transfer by forced convection between a vertical surface and a fluid flowing parallel to it, for conditions of constant surface heat flux. Point values of surface temperatures and heat transfer coefficients were computed using a finite-element solution of the equations for laminar flow and heat transfer within the whole flow domain. In a parallel programme of experimental work, a small electrically-heated heat transfer element was mounted vertically in a tube and a stream of fluid was passed over it in an upwards direction. The surface temperature profiles, measured by means of an array of thermocouples, were in excellent agreement with the computed values. This provided strong evidence of the reliability of both the experimental method and the computational technique.

The experimental method has now been applied to the measurement of temperature profiles for heat transfer by laminar natural convection to fluids covering a very wide range of physical properties (including air, low and high viscosity Newtonian liquids, and shear-thinning polymer solutions). The experimentally determined heat transfer coefficients have been correlated using the standard dimensionless groups (Nusselt, Grashof and Prandtl), suitably modified for application to conditions of constant heat flux, for fluids exhibiting both Newtonian and power-law non-Newtonian rheology.

2. Experimental work

2.1. Apparatus and procedure

Natural convection experiments were carried out with fluid contained in a plastic tank ($430 \times 270 \times 280$ mm tall). The element was suspended centrally in the tank with its surface parallel to the long dimension of the tank, so that the distance between the element and the wall was several times the thickness of the boundary layers. The tank either contained air, or was filled with liquid to within 30 mm of the top, leaving 80 mm of free space above and below the element. The temperature of the bulk fluid was measured using a thermocouple level with the element, but sufficiently far apart for it to be situated outside the boundary layers.

The element itself, the construction of which is described in detail elsewhere [2], consisted of a 90×79 mm support to which was stuck a continuous strip of stainless steel heating foil (0.05 mm thick). The vertical temperature distribution in the foil on one side of the element was measured by means of chromel-alumel thermocouples spot-welded to the back of the foil. Leads for supplying the current were attached to the foil, and separate connections permitted the measurement of the

voltage drop across the element. The heating current was supplied from the AC mains via a step-down transformer, and was controlled by a Variac transformer. The power supplied was calculated from the potential drop across the element and its electrical resistance. The arrangement of the thermocouples on the element is shown in Fig. 1. From a calibration of the resistance over the range of temperatures used in the experiments, it was established that local variations in heat flux over the surface of the foil were not significant. Heat transfer by conduction along the length of the foil accounted for only about 0.1% of the heat transferred to the fluid.

On starting an experiment, the current was set to give the desired heat flux and thermocouple readings were taken as soon as stable conditions had been reached. The time for stabilisation ranged from about 2 min with water to 15 min with viscous liquids. The maximum heat flux used was 15 kW m^{-2} and the corresponding heat input of 0.22 kW resulted in a temperature rise of the bulk fluid of about 1°C over a 30 min period. Thermocouples were read in sequence, starting immediately after the power supply had been switched on, and when the steady state had been reached the temperatures were recorded.

Experiments were carried out with air and three Newtonian liquids and with four non-Newtonian liquids exhibiting power-law behaviour. The rheological properties of the non-Newtonian liquids were measured using a Weissenberg rheometer, as described previously [2]. The relevant physical properties are given in Table 1 for the Newtonian fluids, and in Table 2 for the non-Newtonian fluids. These refer to the initial bulk temperature for the experiments (in all cases $20 \pm 2^\circ\text{C}$). Most other workers have used this basis for physical properties as there is no satisfactory way of taking account of the effect of variations with temperature within the boundary layers.

For each fluid, temperature distributions along the surface of the plate were recorded for a series of heat fluxes. Local values of the film heat transfer coefficient for the fluid were obtained by dividing the heat flux by the temperature difference between the foil and the bulk liquid.

3. Representation and interpretation of experimental results

3.1. Newtonian fluids

For Newtonian fluids, dimensional analysis of heat transfer by natural convection identifies the relevant dimensionless groups as the Nusselt, Prandtl and Grashof numbers. For a point situated a distance x from the bottom of a vertical plate:

$$Nu_x = f(Gr_x, Pr) \quad (1)$$

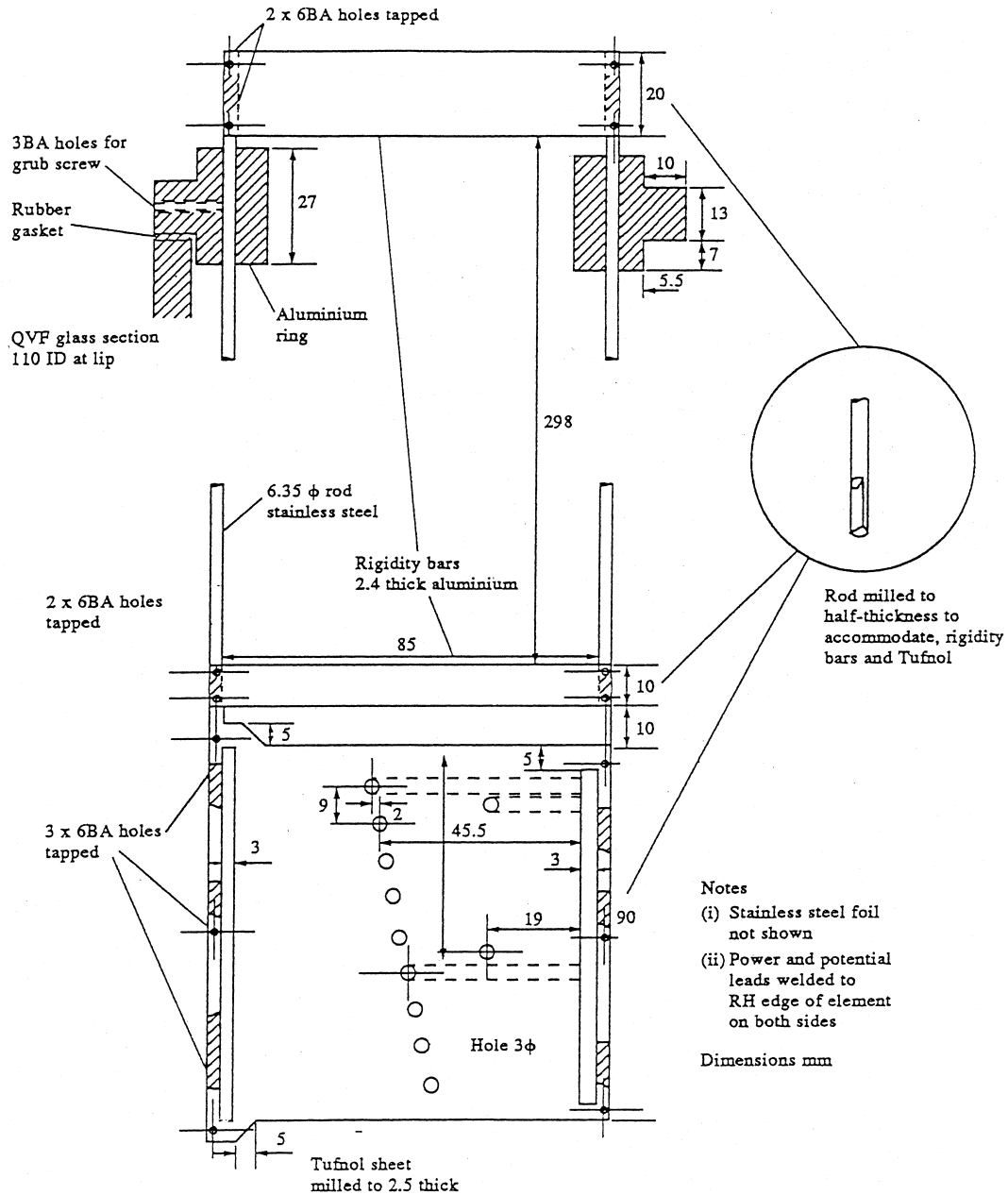


Fig. 1. Arrangements of thermocouples on heat transfer element.

For a constant temperature difference ΔT between the surface and the fluid, the Grashof number is given by:

$$Gr_x = \frac{(\beta g)\Delta T \rho^2 x^3}{\mu^2} \quad (2)$$

By application of classical boundary layer theory and making assumptions concerning the velocity and temperature profiles within the boundary layer, several workers have established that the relation is of the form:

$$Nu_x = c(Gr_x Pr)^{0.25} \quad (3)$$

For instance, Eckert [3], in a theoretical analysis, assumed that the velocity and thermal boundary layer thicknesses were equal and that the velocity and temperature profiles could be represented by parabolic functions. He expressed equation (3) in the form:

$$Nu_x = 0.508 \left[\frac{Pr}{0.952 + Pr} \right]^{0.25} (Gr_x Pr)^{0.25} \quad (4)$$

Table 1
Newtonian fluids—experimental results

	Air	Water	54% Glycerol	84% Glycerol
ρ	1.207	998	1130	1221
μ	0.0000182	0.0010	0.015	0.210
C_p	1006	4182	3276	2707
k	0.0256	0.597	0.408	0.310
β	3.23×10^{-3}	2.07×10^{-4}	4.20×10^{-4}	4.59×10^{-4}
Pr	0.71	7.01	120	1800
$\frac{\rho^2(\beta g)k^3 C_p}{\mu}$	42.8	1.80×10^9	7.8×10^6	2.58×10^6
$(Gr_x^* Pr)_{MAX}$	1.94×10^7	4.26×10^9	1.73×10^{10}	1.94×10^{10}
A	0.472	0.0141	0.0264	0.0520
B	0.833	0.0230	0.0405	0.0876
Confidence limits	$\pm 5.3\%$	$\pm 2.2\%$	$\pm 2.8\%$	$\pm 0.6\%$
$C = A/B$	0.57	0.61	0.65	0.60
C (Churchill)	0.545	0.590	0.623	0.629

$$A = [\mu/(\beta g)\rho^2 C_p k^3]^{0.2}, B = \Delta T/x^{0.2} q_s^{0.8}.$$

$(Gr_x^* Pr)_{MAX}$ = maximum value of the group based on the highest value of the surface heat flux for each material and the distance from the leading edge of the furthest downstream thermocouple (0.0818 m).

All figures in the Table are in basic SI units (kg, m, s, K, N, J, W) and units derived from them.

Thus for Pr of the order of unity (for gases), c is a weak function of Pr . For high values of Pr , c approaches a constant value of 0.508.

Ostrach [4] has also carried out a numerical solution of the boundary layer equations for Pr in the range 0.01–1000 and confirms the value of the exponent in equation (4) as 0.25. There have also been numerous experimental results in support of this value.

For a uniform surface heat flux q_s , temperature (and hence ΔT) is a function of position x . A modified form of the Grashof number may be defined in terms of q_s as opposed to ΔT , to give

$$Gr_x^* = \frac{(\beta g)q_s \rho^2 x^4}{\mu^2 k} \quad (5)$$

and:

$$Gr_x^* Pr = \frac{(\beta g)q_s \rho^2 C_p x^4}{\mu k^2} \quad (6)$$

The counterpart to equation (3) is then:

$$Nu_x^* = C(Gr_x^* Pr)^{0.2} \quad (7)$$

Sparrow [5] has given an analytical solution for this case; it is similar in form to that of Eckert, equation (4), and C is also shown to be a weak function of Pr :

$$Nu_x^* = 0.615 \left[\frac{Pr}{0.8 + Pr} \right]^{0.2} (Gr_x^* Pr)^{0.20}, \quad 0.01 < Pr < 1000 \quad (8)$$

Thus C approaches a constant value of 0.615 at high values of Pr .

Re-arranging equation (7):

$$C \frac{\Delta T}{x^{0.2} q_s^{0.8}} = \left[\frac{\mu}{(\beta g)\rho^2 C_p k^3} \right]^{0.2} \quad (9)$$

Thus, for each value of q_s , a plot of ΔT against $x^{0.2}$ should be linear, if the physical properties of the fluid can be taken as constant over the temperature range involved. In all cases, the experimental points are well represented by a straight line passing through the origin (Figs 2–5). A few experimental points have been omitted for low values of $Gr_x Pr$ ($< ca 10^4$) where the effects of pure thermal conduction become significant and where the assumptions in the boundary layer theory are no longer valid. For very high values ($> ca 10^9$), turbulence may develop in the boundary layer and, in addition, temperature variations in the fluid may be very high; the corresponding points were therefore not taken into account. From equation (9), it is seen that the slopes should be inversely proportional to $q_s^{0.8}$; the values of $\Delta T/x^{0.2} q_s^{0.8}$ were in all cases found to be constant, within the limits of confidence quoted in Table 1. Figures 2–5 show plots for air, water and 54 and 84% (w/w) aqueous glycerol solutions. The linearity of the plots and the near constancy of the values of $\Delta T/x^{0.2} q_s^{0.8}$ implies that neglecting the effect of temperature on the physical properties of the fluid did not introduce any serious error.

For each of the Newtonian fluids, the value of the coefficient C was calculated as A/B where:

$$A = \left[\frac{\mu}{(\beta g)\rho^2 C_p k^3} \right]^{0.2} \quad (10)$$

Table 2
Non-Newtonian fluids—experimental results

	Carbopol 1	Carbopol 2	Carbopol 3	Carbopol 4	Water
n	0.475	0.581	0.599	0.807	1.0
K	0.425	0.713	0.330	0.0564	0.0010
$\frac{n}{3n+2}$	0.139	0.155	0.158	0.182	0.2
$\frac{3n+1}{3n+2}$	0.708	0.733	0.737	0.773	0.8
$\frac{1}{3n+2}$	0.292	0.267	0.263	0.226	0.2
ρ	998	998	998	998	998
C_p	4182	4182	4182	4182	4182
k	0.597	0.597	0.597	0.597	0.597
β	2.07×10^{-4}	2.07×10^{-4}	2.07×10^{-4}	2.07×10^{-4}	2.07×10^{-4}
$(Gr_{NN}^* Pr^n)_{MAX}$	7.69×10^3	9.64×10^3	5.46×10^3	8.55×10^3	4.26×10^9
A_{NN}	$\{C_p^{0.475} \rho^{1.475} K^{-1} k^{1.95} (\beta g)\}^{-0.292}$	$\{C_p^{0.581} \rho^{1.713} K^{-1} k^{2.426} (\beta g)\}^{-0.267}$	$\{C_p^{0.599} \rho^{1.599} K^{-1} k^{2.20} (\beta g)\}^{-0.263}$	$\{C_p^{0.807} \rho^{1.807} K^{-1} k^{2.614} (\beta g)\}^{-0.266}$	$\{C_p \rho^2 K^{-1} k^3 (\beta g)\}^{-0.2}$
	0.1026	0.0781	0.0758	0.0375	0.0141
B_{NN}	$\frac{\Delta T}{x^{0.139} q_s^{0.708}}$	$\frac{\Delta T}{x^{0.155} q_s^{0.733}}$	$\frac{\Delta T}{x^{0.158} q_s^{0.737}}$	$\frac{\Delta T}{x^{0.182} q_s^{0.773}}$	$\frac{\Delta T}{x^{0.2} q_s^{0.8}}$
	0.213	0.155	0.131	0.0585	0.0230
Confidence limits	$\pm 3.9\%$	$\pm 4.9\%$	$\pm 9.8\%$	$\pm 1.9\%$	$\pm 2.2\%$
$C_{NN} = A_{NN}/B_{NN}$	0.49	0.50	0.58	0.64	0.61
C_{NN} (Dale and Emery) (numerically computed)	0.45	0.49	0.50	0.57	0.607
C_{NN} (Tien)	0.73	0.72	0.72	0.70	0.68

$$A_{NN} = \left[\frac{K}{(\beta g) \rho^{1+n} C_p^n k^{1+2n}} \right]^{\frac{1}{3n+2}}, B_{NN} = \frac{\Delta T}{x^{\frac{n}{3n+2}} q_s^{\frac{3n+1}{3n+2}}}$$

$(Gr_{NN}^* Pr^n)_{MAX}$ = maximum value of the group based on the highest surface heat flux for the material and the distance from the leading edge of the furthest downstream thermocouple (0.0818 m).

All figures in the table are in basic SI units (kg, m, s, K, N, J, W) and units derived from them.

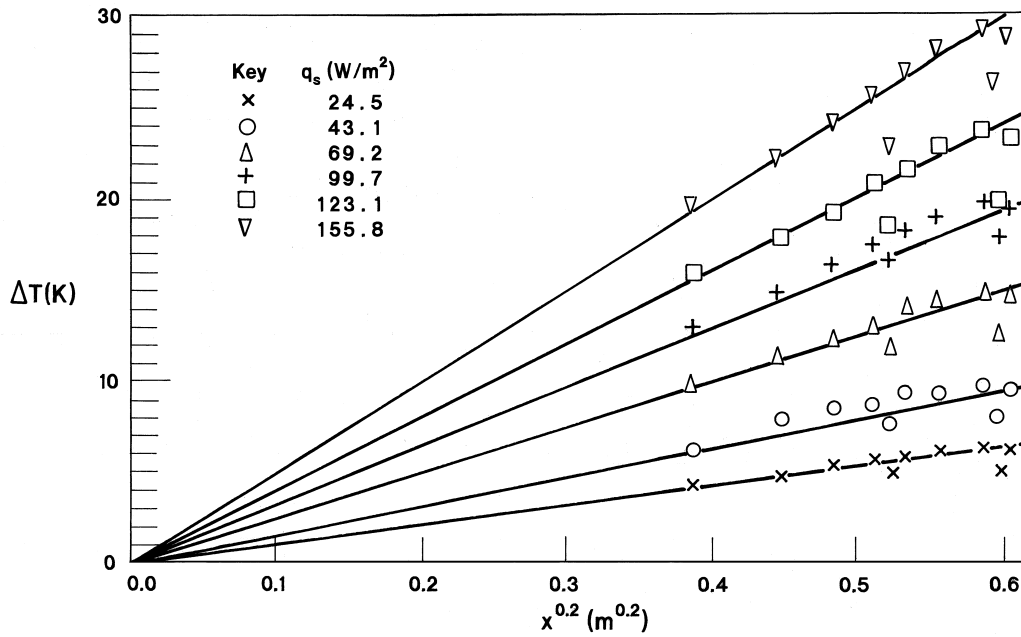


Fig. 2. Experimental results for air.

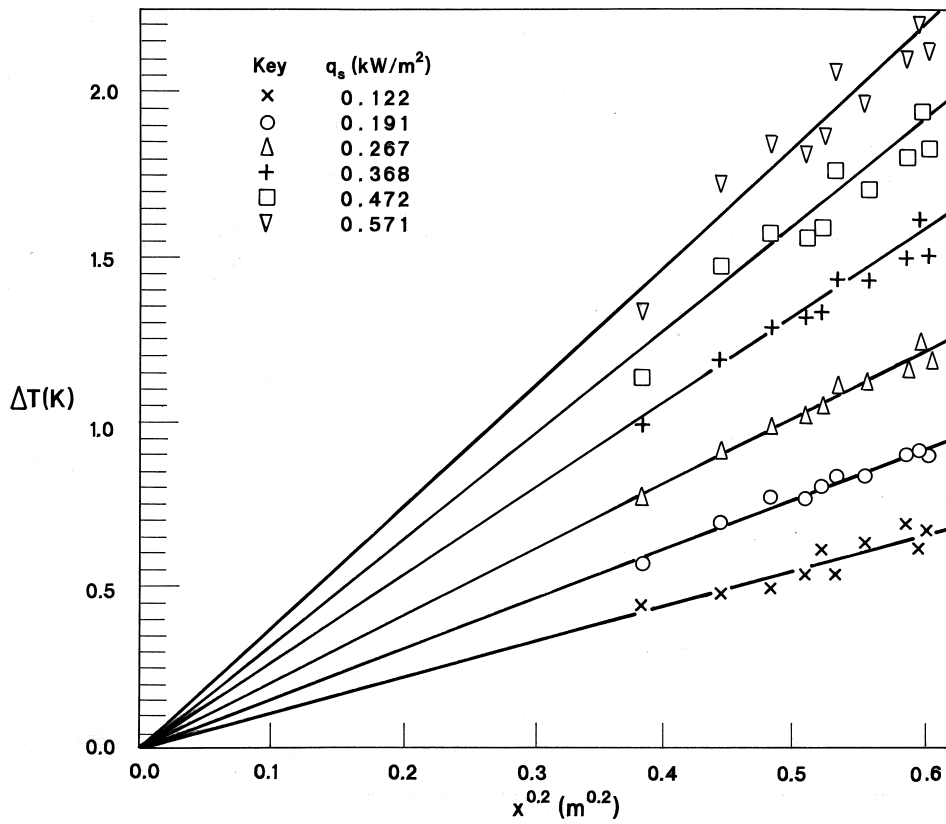


Fig. 3. Experimental results for water.

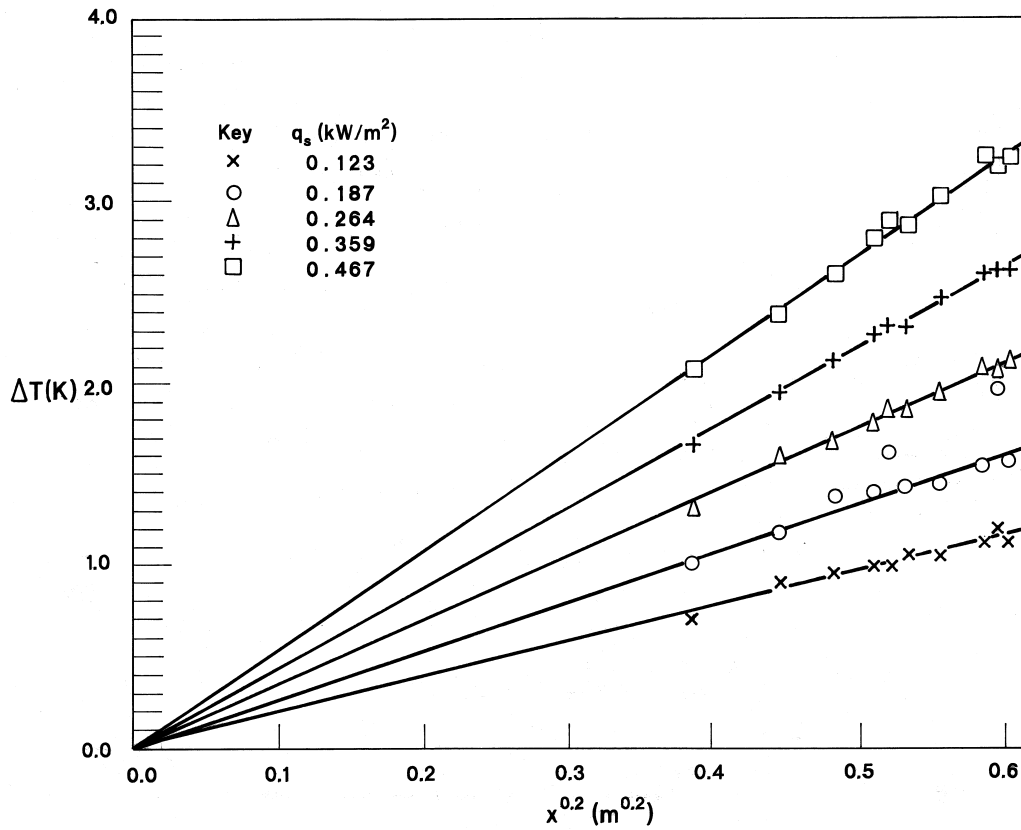


Fig. 4. Experimental results for 54% (w/w) aqueous glycerol.

$$B = \frac{\Delta T}{x^{0.2} q_s^{0.8}} \quad (11)$$

Both the physical properties (including Prandtl numbers) and the experimental results (together with their 95% confidence limits, as given by Student's *t*-test) are summarised in Table 1. The Grashof number is a function of both *x* and *q_s*, having a zero value at the leading edge (*x* = 0), and a maximum value over the experimental domain at the location of the downstream thermocouple and at the highest heat flux. The maximum value of *Gr*Pr* for each fluid is included in Table 1.

3.2. Non-Newtonian fluids

The rheology of each of the non-Newtonian fluids used in the experimental programme was well-represented by the Ostwald-de Waele power-law model $\{\tau = K\dot{\gamma}^n\}$ for shear rates from 0.02 to ca 2 s⁻¹ [2], and none of the fluids was found to exhibit significant elasticity. Inevitably, the use of this model introduces some inaccuracy as all real fluids tend towards Newtonian behaviour as the shear-rate approaches zero. In natural convection, the shear rate is zero at the position of maximum velocity and, in

addition, approaches zero at the extremity of the momentum boundary layer. However, the error is probably small as there do not appear to be problems arising from the use of the power-law model in pipe flow where the shear rate is zero at the centre line. Dale and Emery [6] measured velocity profiles in the fluid near a heated vertical surface for conditions of laminar natural convection and found that the shear rates near the surface were of the order of 1 s⁻¹.

The first requirement is to define appropriate Prandtl and Grashof numbers for power-law fluids, since viscosity is no longer a physical property but is shear-rate dependent. The most satisfactory approach is to replace the kinematic viscosity (μ/ρ) by a new variable with the dimensions L^2T^{-1} . For the power-law fluid, using the consistency coefficient *K* (dimensions $ML^{-1}T^{-2+n}$ and fluid density ρ in combination with the distance *x* from the leading edge, leads to

$$\left(\frac{K}{\rho}\right)^{\frac{1}{2-n}} x^{\frac{2(1-n)}{2-n}}$$

as the appropriate group. On this basis the Prandtl number may be defined as:

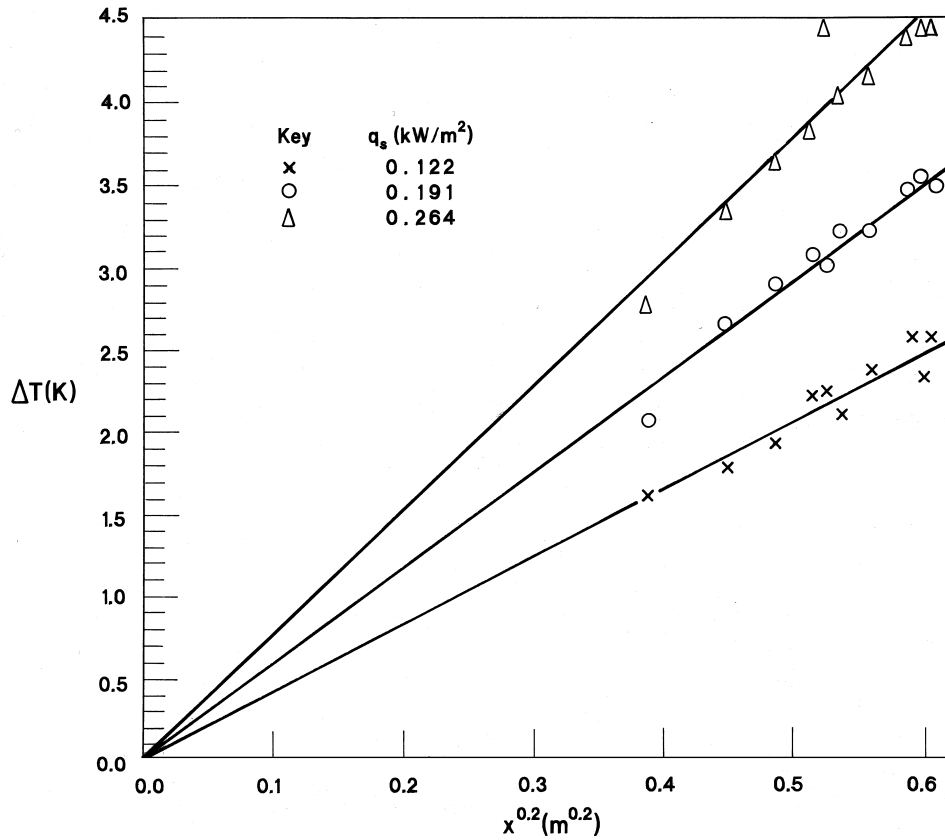


Fig. 5. Experimental results for 84% (w/w) aqueous glycerol.

$$Pr_{NN} = \frac{\left(\frac{K}{\rho}\right)^{\frac{1}{2-n}} x^{\frac{2(1-n)}{2-n}}}{\frac{k}{C_p \rho}} \quad (12)$$

This definition has previously been used in the work of Dale and Emery [6].

The use of x in this way is contentious in that there is little evidence that its use takes proper account of the variation of effective kinematic viscosity within the boundary layer. Similarly, replacing the kinematic viscosity in equation (2), the Grashof number for constant ΔT is given by:

$$Gr_{xNN} = \frac{(\beta g) \Delta T \rho^{\frac{2}{2-n}} x^{\frac{2+n}{2-n}}}{K^{\frac{2}{2-n}}} \quad (13)$$

Acrivos [7] was one of the first to carry out a theoretical analysis of natural convection in power-law fluids, and he obtained a relationship for Nu_x , equivalent to equation (3) for Newtonian fluids. After transforming his Prandtl and Grashof groups to be consistent with the definitions

given by equations (12) and (13), the following relation is obtained:

$$Nu_x = c_{NN} [Gr_{xNN} Pr_{NN}^n]^{\frac{1}{3n+1}} \quad (14)$$

In his derivation, Acrivos assumed that the Prandtl number was much greater than unity. This does not impose any practical limitation as the Prandtl numbers for most non-Newtonian fluids are high. The exponent used in equation (14) is consistent with the results of several experimental studies with power-law fluids, for instance that of Reilly et al. [8].

Equation (14) gives q_s proportional to $\Delta T^{\frac{3n+2}{3n+1}}$.

Again, redefining the Grashof number in terms of q_s for use in the case of constant heat flux (cf. equation (5) for Newtonian fluids):

$$Gr_{xNN}^* = \frac{(\beta g) q_s \rho^{\frac{2}{2-n}} x^{\frac{4}{2-n}}}{K^{\frac{2}{2-n}} k} \quad (15)$$

and:

$$Gr_{xNN}^* Pr_{NN}^n = \frac{(\beta g) q_s \rho^{1+n} C_p^n x^{2(n+1)}}{K k^{1+n}} \quad (16)$$

The dimensionless groups in the case of constant surface heat flux are then related as follows:

$$Nu_x = C_{NN} (Gr_{xNN}^* Pr_{NN}^n)^{\frac{1}{3n+2}} \tag{17}$$

or

$$\frac{q_s x}{k \Delta T} = C_{NN} \left[\frac{(\beta g) q_s \rho^{1+n} C_p^n x^{2(n+1)}}{K k^{n+1}} \right]^{\frac{1}{3n+2}}$$

giving:

$$C_{NN} \frac{\Delta T}{x^{\frac{n}{3n+2}} q_s^{\frac{3n+1}{3n+2}}} = \left[\frac{K}{(\beta g) \rho^{1+n} C_p^n k^{1+2n}} \right]^{\frac{1}{3n+2}} \tag{18}$$

Thus:

$$C_{NN} = A_{NN} / B_{NN} \tag{19}$$

where:

$$A_{NN} = \left[\frac{K}{(\beta g) \rho^{1+n} C_p^n k^{1+2n}} \right]^{\frac{1}{3n+2}} \tag{20}$$

and:

$$B_{NN} = \frac{\Delta T}{x^{\frac{n}{3n+2}} q_s^{\frac{3n+1}{3n+2}}} \tag{21}$$

It will be noted that all of the above equations for power-law fluids reduce to the corresponding equations for Newtonian fluids when $n = 1$.

The results for the non-Newtonian fluids were processed in a similar manner to those for the Newtonian fluids, in this case by plotting ΔT against $x^{\frac{n}{3n+2}}$, checking that the plots were linear, and then measuring the slopes. Results for the four shear-thinning carboxypolymethylenes (Carbopols 1, 2, 3 and 4) are given in Figs 6–9. Again, a few points were omitted, as in the work on Newtonian fluids, when values of the group $Gr_{xNN}^* Pr_{NN}^n$ were either very low or very high. The linearity of the plots was not as good as for the Newtonian fluids, with some appreciable deviations near the leading edge where conventional boundary layer theory may not apply. Furthermore, the inclusion of an arbitrary linear dimension, in this case x in the definition of the Prandtl number, may lead to some discrepancies; this problem was identified earlier in the paper.

The above procedure was repeated for each value of the heat flux q_s and

$$\frac{\Delta T}{x^{\frac{n}{3n+2}} q_s^{\frac{3n+1}{3n+2}}} \quad (= B_{NN})$$

was checked for constancy. C_{NN} was then calculated from equation (19). Values of B_{NN} (and their 95% confidence limits) are given in Table 2. The assumption of constant physical properties again does not seem to lead to serious deviations from the theoretical predictions, except for the few points at very high values of $Gr_{xNN}^* Pr_{NN}^n$ which, as

mentioned above, were omitted, as in the case of Newtonian fluids.

The physical and rheological properties of the non-Newtonian fluids and the experimental results (with their confidence limits) are given in Table 2. For non-Newtonian fluids, both the Prandtl and Grashof numbers are a function of x , and the Grashof number also depends on the heat flux q_s . The maximum value of $Gr_{xNN}^* Pr_{NN}^n$ for each fluid is also calculated in Table 2.

4. Comparison of results with previous work

4.1. Newtonian fluids

In the present work carried out with constant surface heat flux q_s , the value of the coefficient C in equation (7) spanned the range 0.57–0.65 for the various fluids used.

The value for air is 0.57 (Table 1) and this compares with 0.53 calculated from Sparrow’s equation (8). An earlier experimental study by Dotson [9] on natural convection from an electrically heated plate to air gave data points consistent with those calculated from equation (8), though no correlation was given.

For liquids, C in the present work ranges from 0.60 to 0.65 (Table 1), and has a value of 0.61 for water. Other experimental work with water includes that of Vliet and Liu [10] who used a plate (1 × 2 m) covered on one side with electrically heated stainless steel foil, and obtained a value of 0.6 for C . Dale and Emery [6], using a similar technique, have quoted C equal to 0.61, and give a computed value of 0.615. Other computed values for water are 0.61 [5], and 0.59 [11]. Churchill’s correlation of the experimental results of other workers gives a value of 0.59. Thus all of the published values agree quite well. Tien [12] gives a computed value for water (0.68) which is high compared with other workers’ results (see also Section 4.2. dealing with non-Newtonian flow). There is little practical information available for other Newtonian fluids.

For each of the three glycerol–water mixtures used in this study, the computed value according to Sparrow [5] is 0.615 and about 0.62 according to Fujii and Fujii [11]. Fujii et al. [13] carried out experiments both at constant surface temperature and at constant heat flux over the Prandtl number range $2 < Pr < 300$. They measured heat transfer coefficients at the external surface of a large vertical brass tube (1000 mm tall, 60 mm id and 82 mm od); these coefficients were estimated to be not more than 1.3% higher than for a vertical plane surface. Values of C (for constant q_s) were 0.59 for water and approximately 0.62 for the other liquids. Churchill and co-workers [14, 15] have correlated the experimental results of several workers and have given C as a function of Prandtl number. Their values for each of the fluids used in the present work are included in Table 1, and agreement

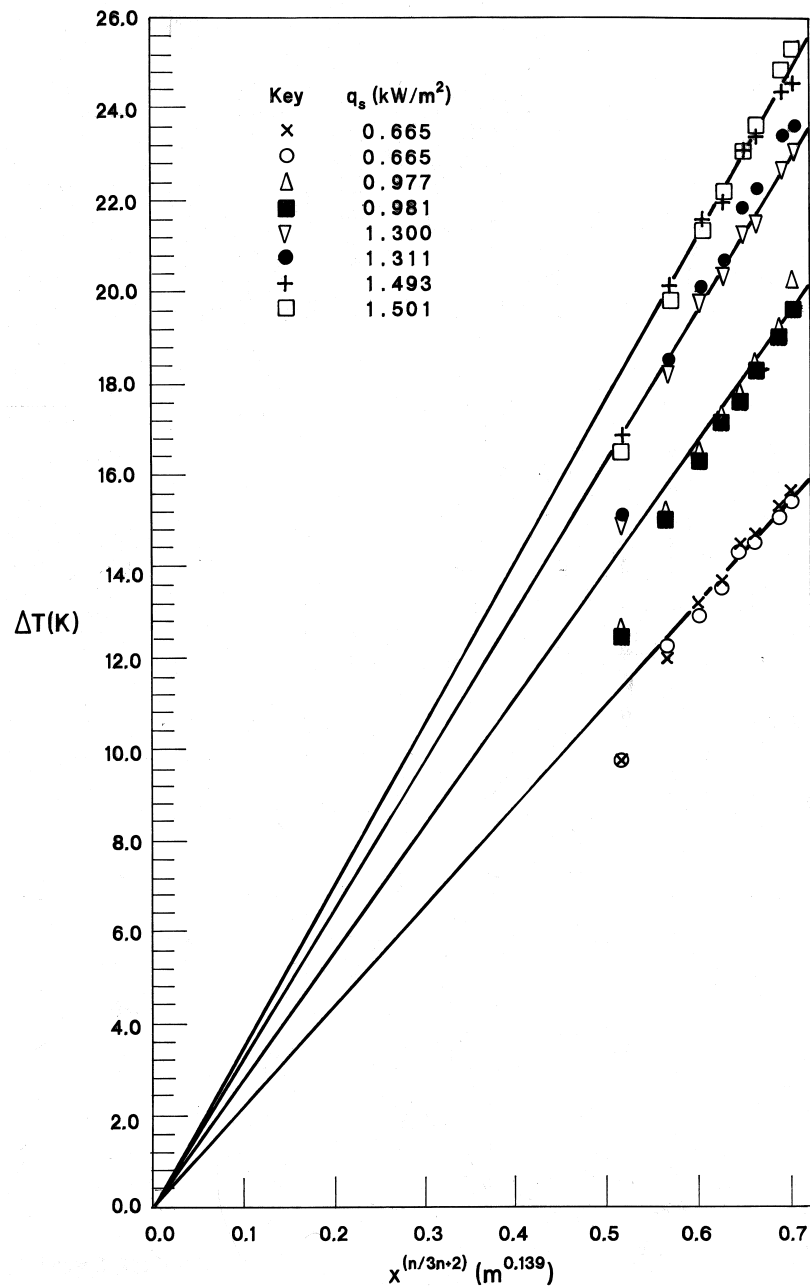


Fig. 6. Experimental results for Carbolpol 1.

between the current work and Churchill's correlations is seen to be good.

For experiments carried out with constant ΔT , the coefficient c in equation (3) for water is given by Reilly et al. [8] as 0.54, as compared with 0.49 calculated from Eckert's equation (4). Pohlhausen [16] gives 0.45 for fluids with Prandtl numbers close to unity (gases). Ostrach [4] carried out a numerical solution of the equations of

motion and compared his calculated values of c with experimental data from the work of Schmidt and Beckmann [17] for air, of Lorenz [18] for oils with Prandtl numbers ranging from 75 to 440, and of Saunders [19] for mercury ($Pr = 0.03$). Mean values of the experimental and calculated values of c were 0.41 for air, 0.42 for the oils, and 0.25 for mercury. More recently, Yamasaki and Nagahashi [20] have measured the heat transfer

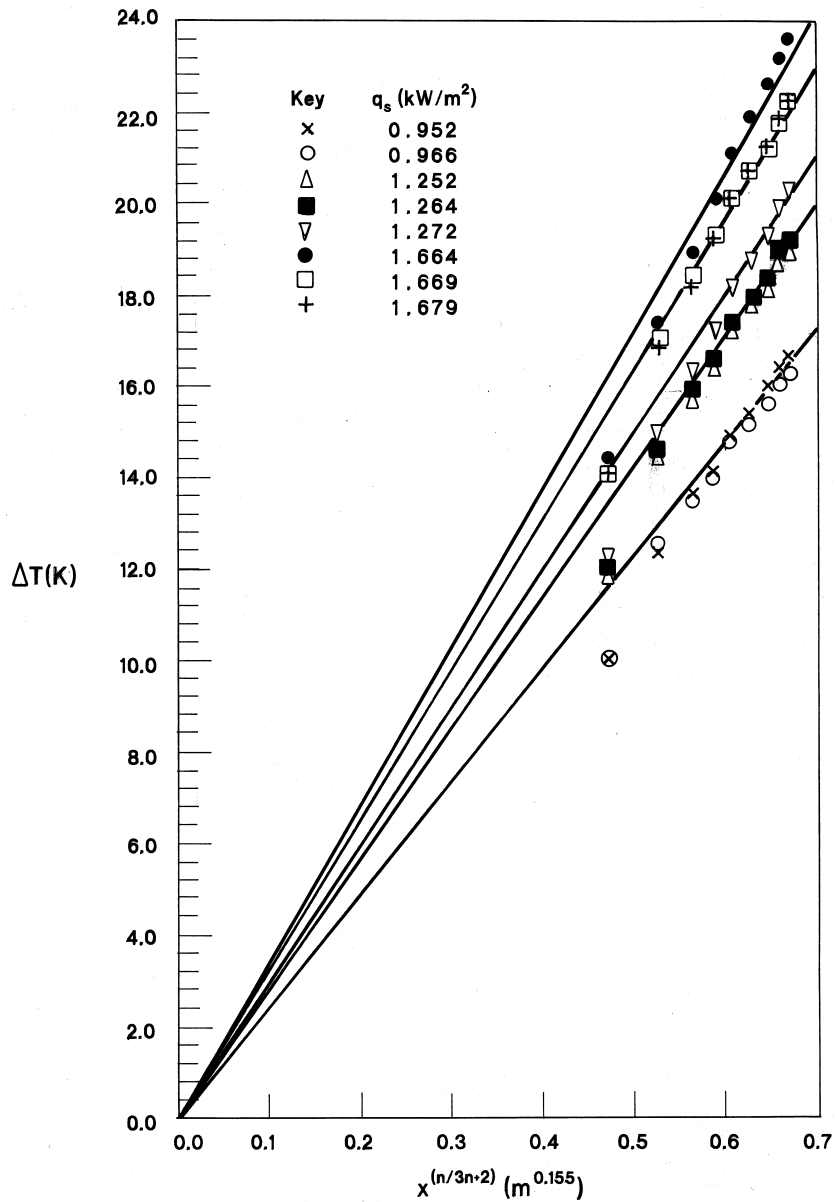


Fig. 7. Experimental results for Carbopol 2.

from a heated plate to air and have reported surprisingly low values of the exponent of the Grashof number (0.178 for $50 < Gr_x < 10^6$). In some of the above investigations, the coefficients which are quoted were average values over the whole surface. It is simply shown by integration that the mean value should be 4/3 times the point value and, where this is so, the appropriate correction has been applied to provide the values quoted above. Values of c for constant ΔT are generally somewhat lower than those for C at constant q_s .

4.2. Non-Newtonian fluids

In the present work, C_{NN} decreases from 0.64 to 0.49 as the degree of shear-thinning increases, as seen from Table 2; however, the value of 0.64 for Carbopol 4 ($n = 0.8$) does look anomalously high in comparison with 0.61 for water ($n = 1$). The nearest comparable work to that carried out in the present investigation is the wide-ranging study of Dale and Emery [6]. They used a technique similar to that employed in the present work, with

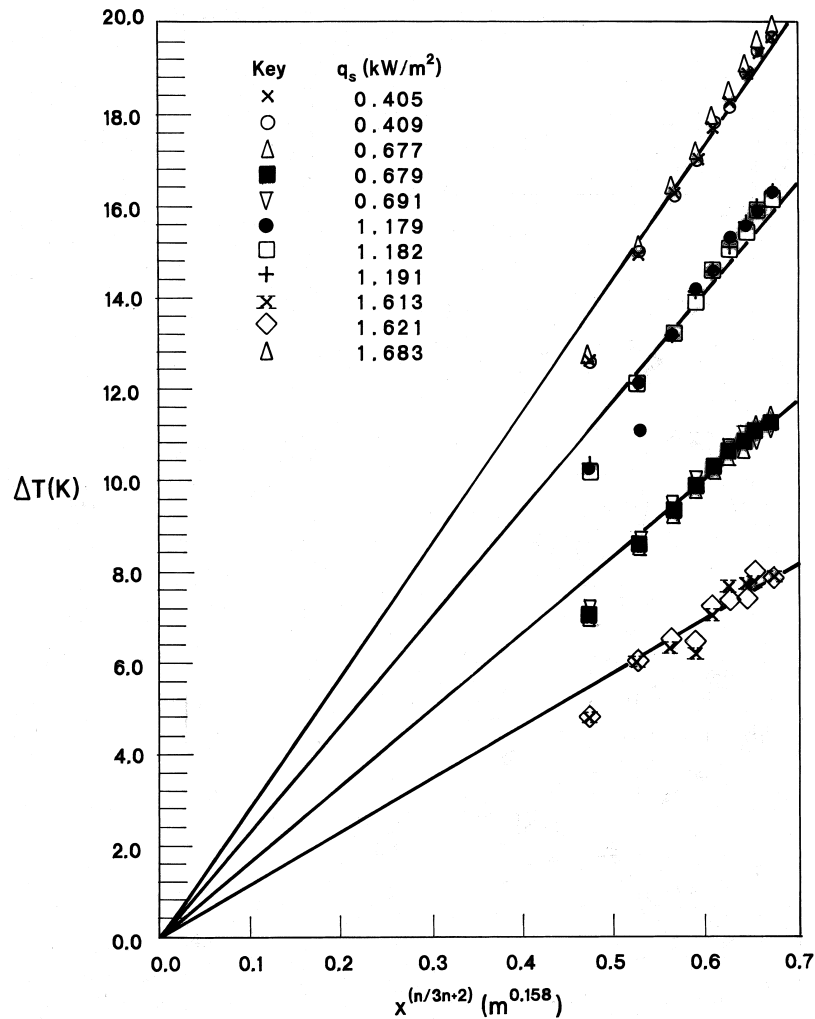


Fig. 8. Experimental results for Carbopol 3.

copper–constantan thermocouples spot-welded to the back of electrically heated foil fixed to an insulating support plate. The scale of their experiments was very much greater with their two heated vertical plates having dimensions of 150×300 mm and 450×600 mm, respectively, though it is not always clear which plate was used in any given experiment. Although surface heat fluxes are not always specified, they are generally in the range 0.2 – 0.8 kW m⁻², comparable with values used in the present work. Water and aqueous solutions of CMC and Carbopol (n -values 0.52 – 0.90) were used in their experiments. They did not, as in the present work, show point values of plate temperature as a function of position and heat flux, but plotted their experimental data points as local Nusselt number against the local value of $G_{s,NN}^* Pr_{NN}^n$ to obtain values of the exponent (which was approximately equal to $1/(3n+2)$) and of the coefficient.

C_{NN} fell within the range 0.51 – 0.66 , but there was no consistent trend with n . Their numerically computed values, based on an exponent of $1/(3n+2)$ are given in Table 2, and are seen to be comparable with those in the present work. Fujii et al. [21], employing the cylindrical brass tube referred to earlier, made measurements at constant surface heat flux for several aqueous polymer solutions. The fluids were found to follow the Sutterby rheological model whose zero shear rate viscosity was used in place of the viscosity in the relations for Newtonian fluids.

Tien [12] has obtained a numerical solution of the boundary layer equations for both shear-thinning and shear-thickening power-law fluids; his relationship is restricted to high values of Prandtl number. He treats conditions of constant temperature difference ΔT and of constant surface heat flux q_s . The total plate length L is

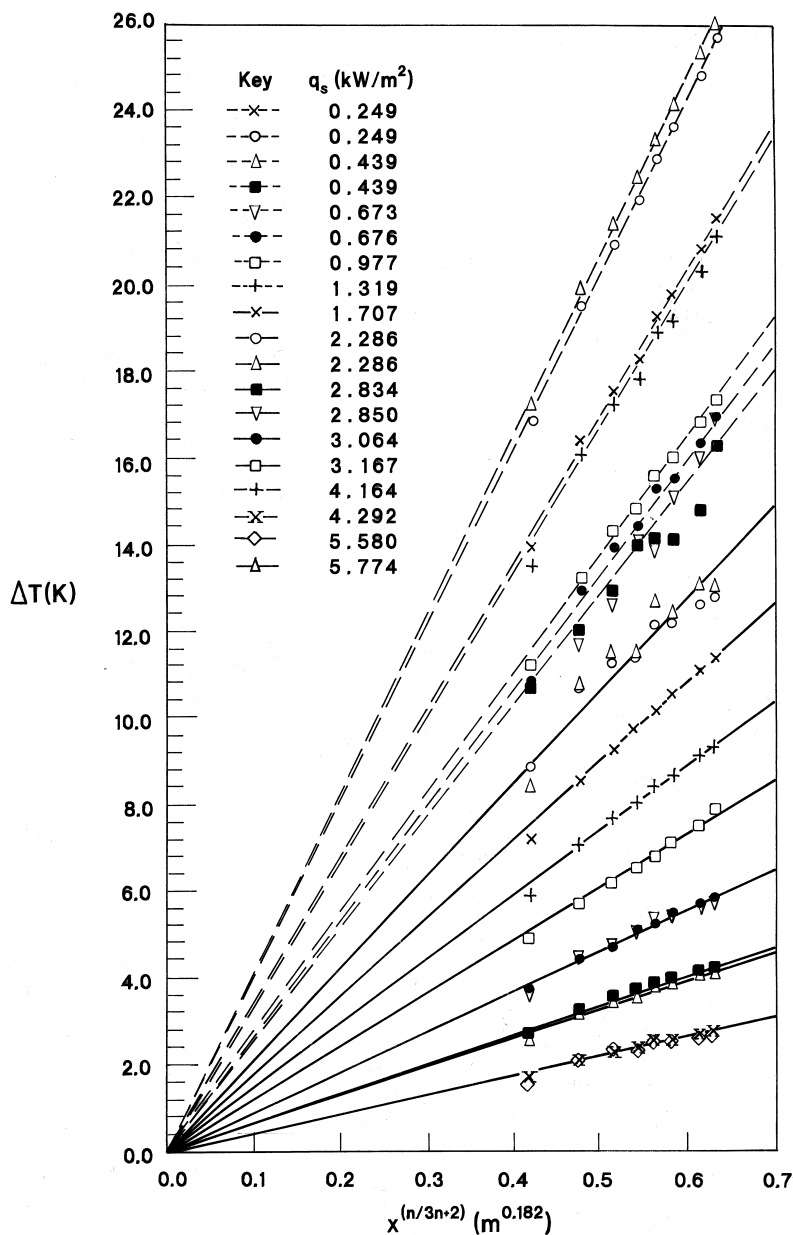


Fig. 9. Experimental results for Carbopol 4.

used as the linear dimension in the dimensionless groups, and it is therefore necessary to include an additional dimensionless ratio x/L in order to obtain the point value of his Nusselt number. For constant q_s , the calculated value of his coefficient M_1 (equivalent to C_{NN} in the present work) ranges from 0.76 to 0.68 as n goes from 0.4 to 1.0. His computed values, which are also given in Table 2, are high compared with both the computed and the experimental values of Dale and Emery and with the present results. Furthermore, they show the reverse trend,

as compared with other workers' results, in that M_1 decreases, rather than increases, as the degree of non-Newtonian behaviour becomes less (n increasing). There is also some confusion in Tien's paper in that he gives two mutually inconsistent forms for his equation, but the most probable interpretation has been placed on this work.

More recently, Huang and Chen [22] have carried out a local similarity solution of the boundary layer equations for constant ΔT and constant q_s , and have shown that

the values of both C_{NN} and c_{NN} are more sensitive to the Prandtl number of the fluid than to the power-law index n . The ratio of C_{NN} to c_{NN} is between 1.25 and 1.3 and, for fluids with properties similar to those used in the present work, C_{NN} is approximately 0.63.

Other experimental studies have been carried out with power-law fluids at constant temperature difference ΔT . For instance, Reilly et al. [8] used Carbopol solutions ($n = 0.72$ and 0.89), and confirmed the value of $1/(3n+1)$ for the exponent in equation (14). Their values of the coefficient, however, related to the average value of the Nusselt number over the surface rather than to the point value.

5. Conclusions

Measurements of heat transfer by laminar natural convection have been made using a small vertical plate (90×79 mm), electrically heated so as to provide a uniform heat flux over the surface. The vertical temperature profile was measured by thermocouples spot-welded to the rear face of the heating foil. The experimental technique had already been validated in work on forced convection in which excellent agreement had been obtained between the temperature profiles measured with this element and those calculated using a finite-element based solution of the complete mass, momentum and energy conservation equations.

Experiments were carried out with a range of Newtonian fluids including air, water and aqueous glycerol solutions, giving a 10 000-fold range of viscosities, a 1000-fold range of densities and a 2500-fold range of Prandtl numbers. In addition, the effects of non-Newtonian behaviour were studied using polymer solutions with power-law rheology, covering n values from 0.48 to 0.81. Thus the effects of physical and rheological properties have been studied over an extremely wide range of values.

The experimental results were analysed by plotting local temperature difference ΔT against $x^{\frac{n}{3n+2}}$ ($x^{0.2}$ for Newtonian liquids). The resulting linear relationships provided confirmation of the theoretical predictions, based on boundary layer theory. By using a range of values of surface heat fluxes q_s , the constancy of the group

$$\frac{\Delta T}{x^{\frac{n}{3n+2}} q_s^{\frac{3n+1}{3n+2}}} \left(\frac{\Delta T}{x^{0.2} q_s^{0.8}} \text{ for Newtonian fluids} \right)$$

was established, thus confirming the predicted dependence on heat flux. Values of the coefficients C for Newtonian fluids, equation (7), and C_{NN} for non-Newtonian fluids, equation (17), were in the range 0.49–0.65, in line with both the computed and experimental results of previous workers.

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